



K18P 1374

Reg. No. :

Name :

First Semester M.Sc. Degree (Reg./Supple./Imp.)

Examination, October 2018

(2014 Admn. Onwards)

PHYSICS

PHY 1C01 : Mathematical Physics – I

Time : 3 Hours

Max. Marks : 60

SECTION – A

Answer both questions, either **(a)** or **(b)**. Each question carries **12** marks.

(2×12=24)

1. a) Find the spectral decomposition of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

OR

- b) Obtain the series solution to the Bessel's equation

$$x^2 y'' + xy' + (x^2 - n^2)y = 0$$

2. a) State and prove Cauchy's residue theorem for complex functions. Using residue theorem, evaluate $\int_{|z|=2} \frac{z}{z^2+1} dz$.

OR

- b) Write Laguerre ordinary differential equation and Laguerre polynomial $L_n(x)$. Obtain Rodrigues' formula for Laguerre polynomials. Deduce first three Laguerre polynomials.

P.T.O.



SECTION – B

Answer **any four** (1 mark for part 'a', 3 marks for part 'b'. 5 marks for part 'c')

(4×9=36)

3. a) Define curl of a vector field.
 b) Express the spherical polar unit vectors in Cartesian unit vectors.
 c) For a fluid flowing through a cylindrical pipe in the z-direction, the velocity fluid is given by $\vec{v} = \hat{z} v(\rho)$. Show that the nonlinear term in Navier-Stoke's equation in hydrodynamics vanishes.

 4. a) Define contravariant tensor.
 b) Obtain the Christoffel symbol of the first kind $|i, j, k| = g_{mk} \Gamma_{ij}^m$ as derivative of the metric tensor.
 c) Prove that every square matrix A can be expressed as sum of two matrices of the form $A = B + iC$ where B and C are Hermitian matrices.

 5. a) Check whether the differential equation $(x^2 - 2xy + y^2) dx + (x^2y - x^3 + y^2) dy$ is exact or not.
 b) Discuss the singular points of the Legendre's equation.
 c) Obtain the indicial equation of $y'' - 2xy' + 2\alpha y = 0$.

 6. a) Check whether $f(z) = e^z$ is analytic or not.
 b) What are different types of singularities of a complex functions ? Give examples in each case.
 c) State and prove Liouville's theorem for an analytic function.

 7. a) What is the relation between beta and gamma functions ?
 b) Find the value of $\int_0^{\infty} e^{-x^4} dx$ in terms of gamma function.
 c) Define double factorial notation. Express the coefficient of nth term of the expansion of $(1 + x)^{1/2}$ in terms of the double factorial notation.

 8. a) Write the first three-Legendre polynomials.
 b) Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.
 c) Define spherical Bessel function. Obtain the expressions for $j_1(x)$ and $j_2(x)$.
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