UGC Minor Research Project

An executive Summary of the report

MRP(S)-871/10-11/KLKA002/UGC-SWRO

Title of the Research Project : Sum Square Graphs

Principal investigator: Dr. Anandavally T. M. K.

The researcher and Germina K.A. introduced a new additive numbering on graphs namely sum square numbering and proved:
The star $K_{1,n}$ is k-sum square for all k; The complete graph K_n is not k-sum square if $n > 3$ and $n \equiv 0, 2, 3, 6, 8, 11 (mod 12)$; if the even cycle C_{2n} is sum square, then n is even, and gave C_4 as a counter example to show that the converse is not true; and Binary trees are strong sum square.
B.D.Acharya defined an integer additive set indexer (IASI) as an injective function $f : V(G) \to 2^{N_0}$ such that the induced function $g_f : E(G) \to 2^{N_0}$ defined by $g_f(uv) = f(u)+f(v)$ is also injective. An IASI is said to be t- uniform if $|g_f(e)| = t, \forall e \in E(G)$. Sum square graph can be considered as a special type of 1-uniform IASI graph.

Results obtained: Paths are k-sum square for all k; If a (p, q)-graph G is k-sum square, then $\sum\limits_{u \in V(G)} f(u)d(u) = qk^2 + q(q-1)k + \frac{1}{6}q(q-1)(2q-1)$; If a (p, q)-graph G is k-sum square, then $q(q-1)(2q-1) \equiv 0 (mod 6)$; If G is r-regular, then $r\Sigma f(u) = qk^2 + q(q-1)k + \frac{1}{6}q(q-1)(2q-1)$; If f is a k-sum square IASI of the complete graph $K_n,\ n > 1$, then $\sum\limits_{u \in V(K_n)} f(u) = \frac{1}{2}nk^2 + \frac{1}{4}n(n-2)(n+1)k + \frac{1}{24}n(n-2)(n+1)(n(n-1)-1)$; If f is a function defined on the vertex set $V(K_n)$ of $K_n,\ n > 1$ such that $g_f(E(K_n)) = \{k^2, (k+1)^2, \ldots, (k + nC_2 - 1)^2\}$, where k is a positive integer, then $\sum\limits_{u \in V(K_n)} f(u)$ is an integer if and only if $n \equiv 0, 2, 3, 6, 8, 11 (mod 12)$. If $n = 12m + 3$ or $n = 12m + 11$, then k and m are of different parities; The odd cycle C_{2n+1} is k-sum square if k and n are of same parity and $k \geq 3n$; If C_{2n+1} is k-sum square, then k and n are of same parity; The cycle C_8 is k-sum square for all k; The cycle C_{12} is k-sum square for all k; Paths are strong sum square; Trees are strong sum square; The complete graph $K_n, n > 2$ is not strong sum square. Even cycles have 2-Uniform IASI; Trees have 2-uniform IASI; graph containing an odd cycle have no 2-uniform IASI; If a graph G has a 2-uniform IASI, then it is a bipartite graph; The complete graph $K_n, n > 2$ have no 2-uniform IASI; The Wheel graph $W_n, n > 4$ have no 2-uniform IASI; The complete bipartite graph $K_{m,n}$ have 2-uniform IASI; Bipartite graphs have 2-uniform IASI; A graph G has a 2-uniform IASI if and only if it is a bipartitie graph.